



# Reconstruction of critical current density profiles from AC loss measurements

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## Abstract

A method of reconstruction of critical current profiles from measurements of a hysteretic AC loss is proposed. As an example, we have applied this approach to study the degradation in the bulk melt-textured HTS sample with a resonance oscillations technique developed earlier for granular HTS samples and now modified to study HTS bulks. The critical current density profile was determined after the degradation in the thin undersurface layer of about 60  $\mu\text{m}$ . © 1998 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

AC losses in any material at the experimental time scale can be classified as frequency dependent or frequency independent losses (viscous or hysteretic losses, respectively). In the low temperature II-type superconductors, the low frequency AC losses in a wide practical range of magnetic fields and temperatures had predominantly hysteretic nature and could be described by the critical state model.

The idea of the critical state model was put forward by Bean back in 1962 [1]. It was the base for a number of models and their modifications [2,3] that provided, in general, a good description of the macroscopic magnetic properties of superconductors.

But just after discovering the high temperature superconductors (HTS), it seemed that the critical state model had lost its meaning due to a great extension of the dynamic region where any magnetic flux variations cannot be considered as quasistatic [4–6]. In terms of AC losses, it means that in certain conditions the viscous losses can surpass the hysteretic ones [6–9]. But today, with the increase of the critical currents in HTS bulks, films and tapes, the critical state model has become actual again.

We have modified the resonance oscillations technique, which was earlier developed for granular HTS investigation [10,11], to study AC losses in quasi-single crystalline HTS bulks. The background and brief description of the technique are given in [12,13]. We have shown that at low AC field of about 10–100 G at liquid nitrogen temperatures the energy loss in HTS has predominantly hysteretic nature. For

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most bulk, HTS samples prepared by the standard melt-textured methods [14,15], the hysteretic part of AC losses can be described by the Bean model with the uniform distribution of a critical current density  $J_c$  in a thin undersurface layer where it can be determined with high accuracy [13]. But if for some reason, such as surface treatment or degradation, the  $J_c$  distribution in the sample becomes nonuniform, the field profiles in HTS volume become more complicated. This appears in the dependencies of the AC loss vs. AC field amplitude first. In this paper, we develop an approach to obtain information about  $J_c$  spatial distribution from AC loss measurements and use it to investigate the superconducting properties degradation in melt-textured HTS bulks with the resonance oscillation technique.

## 2. Experiment and results

In our experiment, a permanent magnet ( $\text{SmCo}_5$ ,  $\varnothing = 6.3 \times 2.3$  mm,  $m = 0.6$  g,  $\mu = 38$  G cm<sup>3</sup>) levitates above the melt-textured HTS sample ( $\varnothing = 32 \times 16$  mm). We use a field cooled case with the initial distance between the permanent magnet and the HTS surface  $x_0 = 4.8$  mm. The system is symmetrical about the vertical  $x$ -axis. A more detailed description of this technique was given in Ref. [13]. The experimental data were obtained for quasi-single crystalline YBCO which was manufactured by the modified melt-textured technique at the IPHT Jena [14].

Fig. 1 represents the experimental dependencies of the inversed  $Q$ -factor of such mechanical system [8] on amplitude  $A$  of the permanent magnet vertical oscillations for one HTS sample just after manufacturing and after half a year. The linear initial dependence  $Q^{-1}(A)$  shows that energy loss per period has two parts:  $W = \alpha A^2 + \beta A^3$  [13], where the first part can be related to viscous losses (that is usual for granular HTS [7,8]) or with losses due to a surface barrier [16], but the second one is well-known hysteretic loss which in a small amplitudes approximation (in assumption that  $J_c(B, x) = \text{const}$ , where  $B$  is magnetic flux density,  $x$  is the distance from the superconducting surface) can be easily obtained from the Bean model:  $W = (c/24\pi^2) (b_{0s}^3/J_c)$ , where  $c$  is the light velocity and  $b_{0s}$  is the AC field amplitude at the superconducting surface. So, the slope of

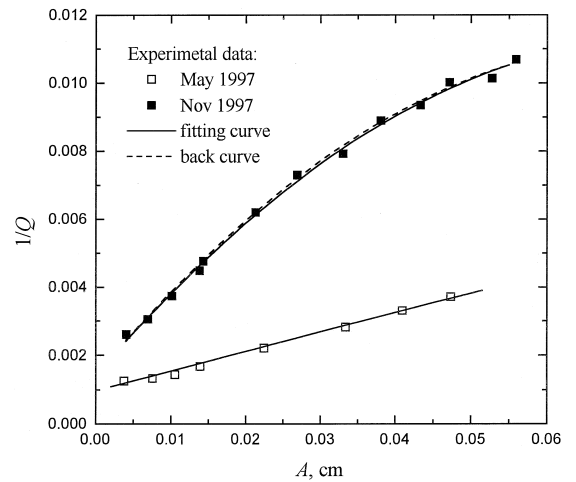


Fig. 1. The experimental dependencies of the inversed  $Q$ -factor on the resonance amplitude of the permanent magnet above the bulk YBCO sample just after manufacturing ( $\square$ ) and in half a year ( $\blacksquare$ ); the dashed curve ('back' curve) represents the dependence obtained back from Eqs. (1), (5), (10) and (12).

the initial  $Q^{-1}(A)$  dependence is determined by  $J_c$  from where its value can be calculated [13]. For the initial sample we have obtained the value  $3.5 \times 10^4$  A/cm<sup>2</sup>. We can also estimate the undersurface  $J_c(x \rightarrow 0)$  of the degraded sample from the initial slope of its  $Q^{-1}(A)$  dependence:  $J_c(0) = 0.79 \times 10^4$  A/cm<sup>2</sup>. Below we investigate a possibility to relate  $J_c(x)$  and  $Q^{-1}(A)$  functions.

## 3. Reconstruction method

We will examine the isothermal response of a nonuniform superconductor to an applied longitudinal AC magnetic field  $B = B_0 + b_{0s} \sin(\omega t)$ , where  $b_{0s} \ll B_0$  and  $B_0 \gg H_{c1}$ , which varies sufficiently slow with time that time derivatives in Maxwell's equations can be neglected. Let us consider a superconductor with the surface lying in the  $y$ - $z$  plane, such that  $x > 0$  represents the superconductor. Then the AC field amplitude inside a superconductor is (see Fig. 2)

$$b_0(x) = b_{0s} - \frac{4\pi}{c} \int_0^x J_c(\xi) d\xi, \quad \text{for } 0 < x < x_p, \quad (1)$$

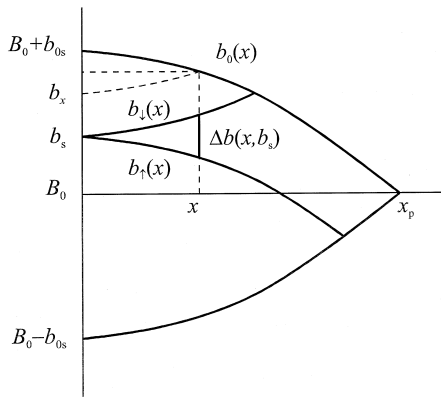


Fig. 2. Magnetic field profiles inside a nonuniform superconductor with the surface lying in the  $y-z$  plane, such that  $x > 0$  represents the superconductor.

where  $x_p$  is an AC field penetration depth that can be determined from the condition  $b_0(x_p) = 0$ . The subscript 's' denotes the superconductor surface. The real field profile inside a superconductor  $b(x, t)$  depends on field change direction and for the given  $b_s = b_{0s} \sin(\omega t)$  we can write  $\Delta b(x, b_s) = b_{\downarrow} - b_{\uparrow}$  (see Fig. 2). Then the energy dissipated per period per unit of surface area is

$$W_s(b_{0s}) = \frac{1}{2\pi} \int_0^{x_p} \int_0^{b_{0s}} \Delta b(b_s, x) db_s dx. \quad (2)$$

For every  $x$  the field  $b_x = 2 b_0(x) - b_{0s}$  can be defined (see Fig. 2) and

$$\Delta b(x, b_s) = 2(b_{0s} - b_0(x)), \quad \text{for } 0 < b_s < b_x, \quad (3)$$

$$\Delta b(x, b_s) = 2(b_{0s} - b_0(x)) \left( 1 - \frac{b_s - b_x}{b_{0s} - b_x} \right), \quad (4)$$

for  $b_x < b_s < b_{0s}$ .

Then from Eqs. (2)–(4),

$$W_s(b_{0s}) = \frac{1}{\pi} \int_0^{x_p} b_0(x) (b_{0s} - b_0(x)) dx. \quad (5)$$

Here  $x_p$  is a function of  $b_{0s}$  and, taking into account that  $b_0(x_p) = 0$ , from Eqs. (1) and (5):

$$\frac{dW_s}{db_{0s}} = \frac{4}{c} \int_0^{x_p} \int_0^x J_c(\xi) d\xi dx, \quad (6)$$

$$\frac{d^2W_s}{db_{0s}^2} = \frac{1}{\pi} b_{0s} \frac{dx_p}{db_{0s}}. \quad (7)$$

At  $x = x_p$ , from Eq. (1) we can obtain

$$\frac{dx_p}{db_{0s}} = \frac{c}{4\pi J_c(x_p)}. \quad (8)$$

and finally, from Eqs. (7) and (8), we obtain the expression for  $J_c$  at a depth  $x_p$  as a function of the experimental data  $W_s(b_{0s})$ :

$$J_c(x_p(b_{0s})) = \frac{c}{4\pi^2} \left( \frac{d^2W_s}{db_{0s}^2} \right)^{-1} b_{0s}. \quad (9)$$

Strictly speaking, Eq. (9) gives the dependence  $J_c(b_{0s})$  at a depth  $x_p(b_{0s})$  that provides some information about the real critical current profile but to obtain  $J_c(x)$  itself in general case numerical calculations are needed.

Let us determine  $J_c(x)$  from the experimental data presented in Fig. 1 for the degraded sample. We will consider the hysteretic part only [13] that can be fitted for this sample by the polynomial  $Q^{-1}(A) = q_1 A - q_2 A^2$ , where  $q_1 = 0.26 \text{ cm}^{-1}$  and  $q_2 = 1.65 \text{ cm}^{-2}$  (see Fig. 1).

The inversed  $Q$ -factor can be expressed as

$$Q^{-1}(A) = \frac{2}{m\omega^2 A^2} \int_0^{R_s} r W_s(b_{0s}(r, A)) dr, \quad (10)$$

where  $m$  is the mass of the magnet,  $\omega$  is the resonance frequency of its vertical oscillations, that slightly depends on  $A$ :  $\omega = \omega_0 - \gamma A$ ,  $\omega_0 = 36.3 \text{ Hz}$ ,  $\gamma = 0.1 \text{ Hz/cm}$ , and  $R_s$  is the radius of the HTS sample. For a flat extremely hard superconductor (see Ref. [12]) at its surface we can write  $b_{0s}(r, A) = 2\theta(r)A$ , where  $\theta(r) = dB_r(r)/dx$ , and  $B_r$  is the component of the permanent magnet field that is parallel to the surface of the sample. For simplicity we assume that  $b_{0s} = 2\theta_{\text{eff}} A$  and use the value  $S W_s$  instead of the integral in (10). Then

$$W_s(b_{0s}(A)) = \frac{\pi m \omega_0^2 b_{0s}^2}{4\theta_{\text{eff}}^2 S} Q^{-1}(b_{0s}(A)). \quad (11)$$

We used the empirical formula  $S = 2\pi k R_s x_0$ , and in our configuration,  $\theta_{\text{eff}} = 0.87 \theta_{\text{max}} = 870 \text{ G/cm}$ ,  $k = 0.94$ . Then, using this assumption, from Eqs. (9) and (11) we obtain  $J_c(x_p(b_{0s})) = J_{c0}/(1 - \varepsilon b_{0s})$  with

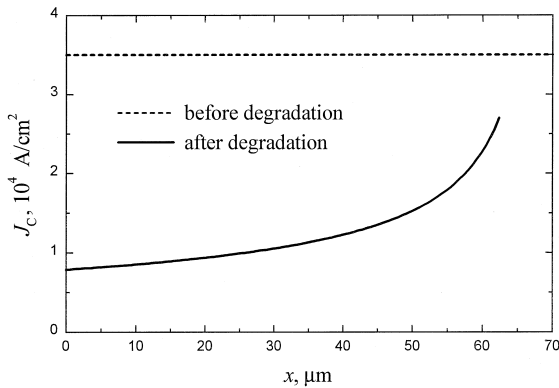


Fig. 3. Critical current density profiles inside the HTS sample before (dashed line) and after (solid line) degradation calculated from AC loss experimental data.

$\varepsilon = q_2/(\theta q_1)$  and, solving differential Eq. (8), finally

$$J_c(x) = \frac{J_{c0}}{\sqrt{1 - px}}, \quad (12)$$

where  $J_{c0} = c\theta_{\text{eff}}^3 S / (3\pi^3 m \omega_0^2 q_1) = 0.79 \times 10^4$  A/cm<sup>2</sup>, and  $p = 8\theta_{\text{eff}}^2 S q_2 / (3\pi^2 m \omega_0^2 q_1^2) = 147$  cm<sup>-1</sup>. The function  $J_c(x)$  is shown in Fig. 3.

The dashed curve ('back' curve) in Fig. 1 represents the dependence obtained back from Eqs. (1), (5) and (12) with real integrating in Eq. (10). The values  $\theta_{\text{eff}}$  and  $k$  were chosen for this curve to coincide with experimental data in two 'points': for the initial slope determined by  $J_{c0}$ , and for  $Q^{-1}(A_{\text{max}})$ . Good coincidence of this back curve with the fitted curve on the hole range gives proof to the feasibility of Eq. (11) instead of Eq. (10).

#### 4. Conclusions

It is necessary to mention that the similar approach for reconstruction of critical current density profiles from AC susceptibility measurements was proposed by Campbell in 1969 [17]. In this paper, we have developed a method of reconstruction of these profiles from hysteretic AC loss measurements. From Eq. (9), the dependence of critical current density at a depth  $x_p$  on AC magnetic field amplitude at the

superconducting surface  $b_{0s}$  can be determined from experimental dependencies of AC losses  $W_s(b_{0s})$  obtained by any experimental technique. Since  $x_p$  is a function of both  $b_{0s}$  and  $J_c(x)$ , the method of getting  $J_c(x)$  from  $J_c(x_p(b_{0s}))$  depends on this function and in general case needs numerical calculations. As an example, we have also shown how the real critical current profile in a thin undersurface layer of melt-textured HTS bulks can be obtained with the simple resonance oscillations technique. The approach proposed can be especially useful to study the surface degradation in HTS bulks and tapes when there is an opportunity to compare the data obtained during the degradation process.

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